The year 2008 has witnessed unprecedented fluctuations in the oil prices. During the first three-quarters, the oil price abruptly increased to $140/bbl, a level that has never been reached before; then because of the global economic crisis, the price dramatically plunged to less than $50/bbl by the end of the year losing more than 64% of the maximum price in less than three months period. The supply of crude oil to the international market oscillated to follow suit according to the law of supply and demand. This behavior affected oil production in all exporting countries. Nonetheless, the demand for crude oil in some developing countries, such as China and India, has increased in the past few years because of the rapid growth in the transportation sector in addition to the absence of viable economic alternatives for fossil fuel. The rapid growth in fuel demand has forced the policy makers worldwide to include uninterrupted crude oil supply as a vital priority in their economic and strategic planning. Even though forecasting should be handled with extreme caution, it is always desirable to look ahead as far as possible to make an intellectual judgment on the future supplies of crude oil. Over the years, accurate prediction of oil production was confronted by fluctuating ecological, economical, and political factors, which imposed many restrictions on its exploration, transportation, and supply and demand. The objective of this study is to develop a forecasting model to predict world crude oil supply with better accuracy than the existing models. Even though our approach originates from Hubbert model, it overcomes the limitations and restrictions associated with the original Hubbert model. As opposed to Hubbert single-cycle model, our model has more than one cycle depending on the historical oil production trend and known oil reserves. The presented method is a viable tool to predict the peak oil production rate and time. The model is simple, accurate, and totally data driven, which allows a continuous updating once new data are available. The analysis of 47 major oil producing countries estimates the world’s ultimate crude oil reserve by 2140 BSTB and the remaining recoverable oil by 1161 BSTB. The world production is estimated to peak in 2014 at a rate of 79 MMSTB/D. OPEC has remaining reserve of 909 BSTB, which is about 78% of the world reserves. OPEC production is expected to peak in 2026 at a rate of 53 MMSTB/D. On the basis of 2005 world crude oil production and current recovery techniques, the world oil reserves are being depleted at an annual rate of 2.1%.

1. Introduction

The demand for crude oil started to accelerate with the invention of the internal combustion engine in the late nineteenth century to become one of the most important commodities traded worldwide. Modern civilization, as it is known to us, heavily depends on crude oil and its byproducts. So far, the industrialized nations have completely taken for granted an uninterrupted supply of cheap hydrocarbon.

One of the prime objectives of the petroleum industry is to provide the modern world with continuous flow of hydrocarbon fluids, oil and gas, while making a profit. Petroleum liquids are exhaustible resources; thus, a good forecasting scheme of oil supply will be crucial to all parties involved in the petroleum business, such as oil companies, financial institutions, public policy planners and makers, and oil exporting and importing countries. Such a model will also help bring stability and security to the crude oil market.

In the past, many researchers have developed different methods to forecast future production of crude oil using either available determinations of ultimate reserves or extrapolation of production history. Decline curve analysis,12 black oil model history matching, and past trend extrapolations are often considered statistical methods of production forecasting.1–8 Econometric models based on physics, economics, technology, and remaining reserves were also used.9–11 In 1995, Skov12 presented a discussion of the various prediction tools used to forecast energy supply and demand. More recently, Monte Carlo simulation models13 and artificial...
intelligence tools, such as fuzzy logic and neural networks, were also used for this purpose.

The Hubbert model is one of the most renowned statistical models for the prediction of oil and gas production. Initially presented in 1956, Hubbert fitted bell-shape curves to cumulative production and discoveries to forecast oil production in the United States (U.S.). He predicted that oil production in the U.S. lower 48 states would ultimately produce about 170 BSTB and that production would peak in 1970. At that time, these predictions were very pessimistic compared to those presented by famous forecasting agencies such as U.S. Geological Survey (USGS) and other forecasters. However, time had shown that Hubbert forecasts were remarkably accurate; oil production did indeed peak in 1970. Since that time, the Hubbert model gained worldwide popularity because of its simplicity and availability of required data and was extensively tested and used to forecast oil production worldwide. Having compared the forecast results of various methods to those of Hubbert, Cleveland and Kaufmann praised the Hubbert model, stating that, despite its lack of theoretical basis, its symmetric parabola predicts production more accurately than regression curves, economic models, or delphi techniques.

Initially, Hubbert obtained his prediction empirically. He fitted historical production data by a normal or Gauss bell-shape curve by making two major assumptions: (i) initially the production rate must start at zero, increase to a maximum, then decline to zero and (ii) the area under the production curve is equal to the ultimate oil recovery as time approaches infinity. Later, Hubbert presented the mathematical foundation for his model where a logistic curve was used to fit cumulative oil production versus time. In the past, several authors have illustrated that Hubbert model with only one full cycle is appropriate to forecast crude oil production for some countries, where the production trend does not exhibit many major fluctuations over time. However, recent studies have shown that most worldwide oil producing countries display more than one Hubbert production cycle. Therefore, the application of the conventional Hubbert model to these countries is inappropriate and does not yield good forecasting results. The additional production cycles are apparently the result of many factors, reflecting the state-of-the-art technological evolution in the oil industry, government regulations, economic conditions, and political events. The single-cycle Hubbert model does not consider the effects of these factors. Al-Fattah and Startzmann were the first to modify Hubbert model to account for the various factors affecting production. They introduced what is called “multicycle Hubbert” approach to forecast world natural gas production. Their model was highly accurate in generating past production history and providing accurate forecast.

The objective of this study is to apply the multicycle Hubbert approach to forecast the world oil production after evaluating production trends of 47 major oil producing countries, which virtually supply most of the world conventional crude oil. Each country will have its own prediction model; then, the world model will be determined by combining the models of all countries. In this manner, the world model will provide a realistic production forecast because it covers the combined performances of all producing countries. Estimates of the ultimate recovery, remaining reserves, and future production of crude oil for each country will be provided. Furthermore, a separate model for OPEC countries will be presented and the organization impact on the world oil production and reserves will be illustrated.

2. Methodology

2.1. Mathematical Formulation. Al-Fattah and Startzmann presented the multicycle Hubbert equation as

\[ q(t) = \sum_{i=1}^{k} q(t)_{i} = \sum_{i=1}^{k} \frac{4(q_{\text{max}})}{[1 + e^{-a(t-t_{\text{max}})}]^2} \]

where \( k \) is the total number of production cycles, \( q_{\text{max}} \) and \( t_{\text{max}} \) are the peak oil production rate of each cycle and its corresponding time, respectively, and \( a \) is a constant. The parameters involved in eq 1 can be calculated using a nonlinear least-squares computation technique.

The multicycle Hubbert model as represented by eq 1 has several advantages over the conventional single-cycle Hubbert equation. The most important advantages are summarized as follows:

1. It is derived from physical and mathematical concepts.
2. It uses readily available historical production data.
3. It can match, with good accuracy, historical data fluctuations influenced by technological, economical, and political events.
4. The predictions can be easily regenerated when new production data are available.
5. Equation 1 has three unknown parameters; whereas the conventional Hubbert equation has four unknown variables. This fact renders the historical production data easier to model.

Each production cycle of the multicycle model has its own value of ultimate recovery, \( Q_{\text{max}} \), which can be calculated from the
The total ultimate recovery, \( N_{pa} \), is then computed by adding the ultimate recovery from each production cycle as follows:

\[
N_{pa} = \sum_{i=1}^{k} 4\left(\frac{q_{max}}{a}\right)_{i}
\]  

The logistic curve representing the cumulative production of the multicyclic model is expressed as

\[
Q = \sum_{i=1}^{k} \left[ 1 + e^{-\left(\alpha t - \tau_{max}\right)} \right]_{i}
\]  

The future recoverable oil, \( N_{FR} \), is determined by subtracting the cumulative production (eq 4) from the ultimate recovery (eq 3).

\[
N_{FR} = N_{pa} - Q
\]  

### 2.2. Peak Oil Production

Three different analytical procedures can be used to calculate the peak oil production. These procedures are summarized as follows: (1) correlating back-dated discovery data with production data with shifted time lag, (2) using known ultimate recovery; if it is unknown, it can be calculated by adding the cumulative production and proven reserves, and (3) using the method of inflection points.

It is always desirable to use more than one of the above procedures to improve the confidence level of the prediction. A detailed derivation of the method of inflection points is presented in the Appendix.

### 2.3. Production Data Acquisition

Unlike other statistical forecasting methods, the multicyclic Hubbert model uses readily available historical production data. The data used in this investigation were collected from many reliable sources, such as the Twentieth Century Petroleum Statistics,36 Oil and Gas Journal37 (OGJ) database, World Oil Journal38 (WOJ), Energy Information Administration39 (EIA), and OPEC40 official Internet database Web site. The data collected for each country consist of annual oil production from the time data are published up to year 2005 and recently updated reserves. Production data of 47 major producing countries virtually holding most of the global oil reserves were used to develop the forecasting models.

### 2.4. Data Analysis Procedure

A total of 47 forecasting models, each representing one of the investigated countries, were developed. Historical production data for each country were scrutinized, and the number of production cycles was determined on the basis of initial data examination. At a later stage of the modeling process, additional cycles where sometimes required to have a more accurate fit. The multicyclic model as presented by eq 1 was solved using a nonlinear least-squares numerical computation technique with initial parameters.

Each production cycle has three parameters, \( a \), \( q_{max} \), and \( l_{max} \) that need to be calculated. For example, if the historical production data of a certain country exhibit two production cycles, then six parameters would be required for the model. The optimum values of the parameters were obtained by minimizing the root-mean-square of the errors, RMSE, of the production rates. RMSE is a measure of the data dispersion around zero deviation. It is defined as

\[
RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (q_{act} - q_{cal})^2}
\]

where \( q_{act} \) and \( q_{cal} \) represent the actual and calculated production rates, respectively, and \( n \) is the number of data points.

Once the optimal values of the model parameters for each production cycle are determined, the ultimate recovery for each production cycle is computed using eq 2. The total ultimate recovery is then calculated by summing up the ultimate recoveries of all production cycles as illustrated in eq 3. The cumulative production is computed using eq 4 by adding the annual crude oil production from previous years.

### 2.5. Model Assessment Criterion

The goodness of fit of the different countries’ models is appraised using the coefficient of variation factor, CV, which is defined as the ratio of the root mean squares of errors, RMSE, of each country’s model to its peak production rate

\[
CV = \frac{RMSE}{q_{max}}
\]

The values of CV are sorted and the percentile ranking probabilities are then calculated using the following equation:

\[
F(CV) = \frac{r - 0.5}{n}
\]

where \( r \) is the country’s rank in the goodness of fit of the production model and \( n \) is the total number of countries.

The goodness of fit of all models is arbitrarily classified as excellent, very good, good, or poor. A certain country’s model has an excellent fit if its coefficient of variation, CV, is less than the mean, \( \mu \), of CV of all countries minus their standard deviation, \( \sigma \), \( CV < (\mu - \sigma) \). This criterion indicates that the model accurately fits the data. If the CV of the country’s model falls between the mean minus one standard deviation and the mean plus one standard deviation, \( \mu < CV < (\mu + \sigma) \), this implies that the fit is very good. If the CV is between the mean plus one standard deviation and the mean plus three standard deviations, \( \mu + \sigma < CV < (\mu + 3\sigma) \), this indicates that the fit is good. If the CV is greater than or equal to the mean plus three standard deviations, \( CV \geq (\mu + 3\sigma) \), this designates that the model is poor, and the generated values have a great deviation from the actual data.

### 3. Analysis of Prediction Results

This section presents the analysis of production forecasts for 47 most influential countries around the world in terms of ultimate crude oil reserves. We essentially investigated every country around the globe that has a proven oil reserves higher than 0.468 BSTB. Furthermore, we classified the countries into two major categories: (1) OPEC countries and (2) non-OPEC countries. First, we will present prediction results for each category; then, we will aggregate all the production data and provide an analysis forecast for the entire world. Moreover, we will select Saudi Arabia from the OPEC countries and the U.S. from the non-OPEC countries as the most prominent countries in each category to illustrate the multicyclic approach. Because of the large number of investigated countries and the length limitation of the paper, it is not possible to display the prediction model of each country in graphical form; thus, only those countries having a prominent impact on the supply and demand of crude oil will have their prediction models presented in graphical form, the results of the remaining countries will be given in table form.

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3.1. **OPEC Countries.** OPEC comprises Algeria, Angola, Indonesia, Iran, Iraq, Kuwait, Libya, Nigeria, Qatar, Saudi Arabia, United Arab Emirates (UAE), and Venezuela. Eventhough Angola joined the organization in 2007; its production will be included in the analysis since the forecasting period will extend beyond 2007. Before presenting the prediction results, it is important to give a brief historical background about the major events in OPEC history and its influence on global crude oil production, oil price, and policy making.

OPEC's actual production was mainly unrestricted until the 1973 Arab oil embargo. After that, the production flattened at around 30 MMSTB/D until 1979, when the Iranian revolution occurred. The average price of the U.S. crude oil increased from $12.64/bbl in 1979 to $21.59/bbl in 1980 and then to $31.77/bbl in 1981 because of a shortage in the crude oil supply during the Iran–Iraq war.41,42 OPEC countries attempted to maintain high oil price by controlling the oil production when the demand fell, as a consequence of this strategy, the oil production dropped from 30 MMSTB/D in 1979 to about 16 MMSTB/D in 1985. This move was supposed to raise the crude oil prices; however, the reduction in OPEC production was counterbalanced by an increase in production of non-OPEC countries, such as Mexico, China, U.S. (Alaskan fields), Canada, and Western European countries (North Sea fields), along with few OPEC countries that did not abide by their production quota. This behavior resulted in a sharp decrease and fluctuation in the oil price. The average price of the U.S. crude oil decreased from $24.09/bbl in 1985 to $12.51/bbl in 1986. Up to 2005, OPEC continues the post-1985 production trend where the production increased from 16 MMSTB/D to 31.1 MMSTB/D. As of 2005, OPEC’s share of the world crude oil production was about 45%.

Figure 1 displays the actual and predicted oil production for Saudi Arabia. It is well-known that Saudi Arabia has the largest oil reserve in the world, and it will remain by far the highest crude oil producer for years to come. Consequently, there is no doubt that it will continue to be a major key player in balancing the world crude oil market, not only because of its high production capability but also because of its spare reserves and moderate energy policy. The forecasting model illustrated in Figure 1 indicates that Saudi Arabia’s production rate is anticipated to increase from 9.4 MMSTB/D in 2005 to 12.2 MMSTB/D in 2015, and it is estimated to peak in 2027, at a production rate of 14 MMSTB/D. The ultimate oil recovery ($N_{pa}$) predicted from the model using eq 3 is about 370.5 BSTB. The oil produced by 2005, calculated by eq 4, is 107.7 BSTB. Thus, our proposed model predicts an ultimate reserve, $N_{FR}$, of 262.8 BSTB (eq 5). This value closely matches the 262.3 BSTB reserve value published by the EIA39 and OPEC.40

The prediction models of the other OPEC countries are displayed in Figures 2–12. This investigation shows that Iran (Figure 2) and Indonesia (Figure 3) have already reached their production peaks in 1974 and 1977, respectively. Angola (Figure 4) and Algeria (Figure 5) will peak in 2010 and 2012, respectively. The other OPEC countries will peak between 2017 and 2036. Iraq (Figure 6) is expected to be the last OPEC country to reach its peak production of 8.5 MMSTB/D in 2036 because of its huge oil reserves and the political events that affect its production capability. Moreover, some OPEC countries other than Saudi Arabia, such as Iraq, Iran, Kuwait (Figure 7), UAE (Figure 8), Venezuela (Figure 9), Nigeria (Figure 10), and Libya (Figure 11) show a promising future oil production increase. Moreover, this study shows that Iran has the potential to boost its production to the prerevolution level of about 6 MMSTB/D.

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(42) Talabani, S. Presented at SPE Middle East Oil and Gas Show and Conference, Manama, Bahrain, March 2005; Paper SPE 92888.
According to our projection, this may occur around year 2032.

The oil production model of all OPEC countries is illustrated in Figure 13. This model was generated by combining the multicyclic Hubbert models of all 12 OPEC countries. Figure 13 shows an excellent match between the actual and predicted production rates. The model indicates that OPEC crude oil production will peak at 53 MMSTB/D in 2026. The production is expected to decrease to 29 MMSTB/D by 2050. At that time, the production will come from few OPEC countries, mainly, Saudi Arabia, Iraq, Iran, Kuwait, UAE, and Venezuela. Other OPEC countries will struggle to lift output, with production dropping in Qatar (Figure 12), Algeria, and Indonesia. This indicates that several OPEC members such as Indonesia, Angola, Algeria, and Qatar will soon convert from exporters to potential importers.

OPEC production model also fits the cumulative production data very well as illustrated in Figure 14. Our proposed approach shows that OPEC ultimate conventional crude oil reserves and future recoverable oil, that is, oil remaining to
be produced, are about 1321 BSTB and 909 BSTB, respectively. Production in OPEC countries, especially in the Middle East, is expected to increase more rapidly than in other regions because their resources are much larger and their production costs are generally lower. These projections are broadly adequate with proven reserves, since OPEC oil prices, production policies, and national policies on developing reserves are extremely uncertain. On the basis of the 2005 OPEC production of 31.1 MMSTB/D and the remaining reserves, OPEC crude oil reserves are being depleted at an annual rate of 1.25%.

3.2. Non-OPEC Countries. This category consists of 35 countries. The proven reserves of these countries range from 0.468 BSTB in case of Romania to 74.436 BSTB in case of Russia. In the subsequent discussion, we will only focus on the main producing countries that have and will have, for sometime to come, major influence on the supply and demand of crude oil. Consequently, we will inspect crude oil production from U.S., Canada, Mexico, China, Brazil, Norway, India, United Kingdom (U.K.), Russia, and Kazakhstan.

United States crude oil production reached its peak of 9.66 MMSTB/D in 1970, after that time the production started to steadily decline. This trend was reversed by the discovery and production from the Alaskan fields. The contribution of Alaskan production increased from about 0.5 MMSTB/D in 1977 to about 3 MMSTB/D in 1985. This resulted in a short-term production increase in the U.S. Figure 15 displays the multicyclic model as applied to the U.S. historical production data. The model fits the daily production data remarkably well, as well as the cumulative production data as illustrated in the figure. Moreover, the multicyclic model predicts that the total ultimate oil recovery ($N_{pa}$) is 217.7 BSTB (eq 3); 188 BSTB of this volume has been produced by 2005, as calculated by eq 4. Thus, the oil remaining to be produced, as determined from the model (eq 5), is 29.7 BSTB. This value is in excellent agreement with the published reserve value of 29.9 BSTB. Our calculation shows that the U.S. crude oil reserve is being depleted at an annual rate of 6.3%.

Figure 10. Nigeria crude oil production model.

Figure 11. Libya crude oil production model.

Figure 12. Qatar crude oil production model.

Figure 13. OPEC crude oil production model.

Figure 14. OPEC crude oil production versus cumulative production.
The models of Kazakhstan, Brazil, and India are displayed in Figures 16–18, respectively. Kazakhstan (Figure 16) has a good potential to be a key provider of crude oil to the international market for some time to come because of its considerable crude oil reserve of about 39.62 BSTB. As of 2005, the production rate of this country was around 1 MMSTB/D. The multicyclic model shows that Kazakhstan has the capability to rapidly lift up its oil production to reach its peak of 5.56 MMSTB/D in 2020.

Brazil’s model is displayed in Figure 17. Our prediction indicates that Brazil, which has a proven reserve of about 12 BSTB, is on the verge of peak production in 2010 with a production rate of about 2 MMSTB/D. Figure 18 shows that the Indian historical oil production had several sharp fluctuations before reaching a production plateau in 1995 at a rate of 0.67 MMSTB/D. The multicyclic model predicts that the Indian oil production will peak in 2015 at a production rate of 0.94 MMSTB/D.

As of year 2009, our study shows that Russia (Figure 19), Mexico (Figure 20), Canada (Figure 21), Norway (Figure 22), U.K. (Figure 23), and China (Figure 24) have already reached their peak production times as presented in Table 1. Both Norway and U.K. production trends show steep decline in oil production since 2000. Mexico’s production behavior sharply increased from 0.42 MMSTB/D in 1970 to peak in 2004 at 3.4 MMSTB/D. Since 1970, China’s production trend has steadily increased from 0.2 MMSTB/D to reach its peak of 3.82 MMSTB/D in 2009. Our forecast demonstrates that out of the six previously listed countries only Russia is able to substantially increase its production to peak at 11.2 MMSTB/D in 2009.

The model of all 35 non-OPEC countries is displayed in Figure 25. The figure shows an excellent match between the historical and predicted data. The model reveals that non-OPEC countries have already reached their peak production rate of 39.6 MMSTB/D in 2006. The cumulative production data predicted by the model nicely capture the actual production rates, as illustrated in Figure 26. This figure also shows that the ultimate crude oil reserve of non-OPEC
countries is about 819 BSTB. According to our projection, starting from 2005 there are about 252 BSTB remaining to be produced. This implies that non-OPEC crude oil reserves are being depleted at a rate of 5.6% per year.

3.3. Total World Forecast. The world model comprising all 47 countries, OPEC and non-OPEC, is displayed in Figure 27. The model indicates an excellent match with the actual data for both production rate and cumulative production. As shown in the figure, the model reveals that the world production will peak in 2014 at a production rate of 79 MMSTB/D, and then it will start declining to reach about 30 MMSTB/D in 2050. Figure 28 illustrates the world actual production rate versus predicted cumulative production. Our model estimates the world ultimate reserve at 2140 BSTB of which 979 BSTB has already been produced by 2005. Thus, the remaining oil to be produced is about 1161 BSTB. OPEC countries hold about 78% of the world remaining reserves. On the basis of 2005 world crude oil production and recovery techniques, the world crude oil reserves are being depleted at an annual rate of 2.1%. The distribution of the ultimate and future recoverable oil of the top 11 countries in the world is displayed in Figure 29. These countries contribute to 76% of the total world ultimate crude oil.

It is important to mention that the 2009 total world oil production is about 85 MMSTB/D, our model predicts a production rate of 74 MMSTB/D, a difference of about 11 MMSTB/D. This is basically because of three facts: First, we did not use oil production from every oil producing country around the world simply because the production rates of some countries are irrelevant compared to other countries and do not have major impact on the global crude oil supply and demand. Second, our predictions are solely based on conventional crude oil production and proven reserves, that is, crude oil production from shale oil and oil sands is not included in the forecast. Shale oil and oil sands production highly fluctuates with crude oil prices, global economy, and available technology. Third, we only used confirmed conventional crude oil reserves; from time-to-time, some countries provide information
on suspected new discoveries and accordingly update their reserves based on these speculations. Unless confirmed, the new reserves overestimate the future forecast.

3.4. Statistical Analysis. In general, our forecasting models accurately match the actual oil production of all investigated countries. The prediction results along with the corresponding error analysis results of RMSE and CV values for all countries are presented in Table 1. Figure 30 displays a semilog plot of CV versus the percentiles rankings. The figure shows that the CVs are log-normally distributed around their mean, \( \mu = 2.85\% \) and standard deviation, \( \sigma \), of 1.41 (\( \sigma = 1.41\% \)). According to the model assessment criterion set in this study, the models of eight countries out of the 47 investigated countries have excellent fit with the actual published data (\( CV < 1.44\)). The goodness of fit can be also illustrated by inspecting the cross plot of the actual versus predicted reserves (Figure 31), which shows that the plotted points are tightly grouped along the unit slope line indicating excellent correlation between the actual and predicted values.

3.5. Model Validation. The accuracy of the prediction forecast is illustrated by comparing the actual production of the last three years (2006–2008) to the predicted production from the multicyclic models of OPEC, non-OPEC, and total world. The production data of the last three years were not used in the initial modeling process; they were spared for validating the forecasting models. This is an important step to test the reliability and effectiveness of the prediction models. The comparison is displayed in Figures 32–34, where the actual production rates are plotted on the same graphs as the model predicted rates. It is quite obvious in all three figures that the actual rates align along the solid line of the predicted production indicating that the multicyclic model highly duplicates the real production data. Moreover,
we calculated the RMSE for each country for the last three years. The models of OPEC countries have minimum, maximum, and median RMSE values of 0.068 MMSTB/D, 0.821 MMSTB/D, and 0.256 MMSTB/D, respectively; whereas, the non-OPEC countries have a minimum RMSE value of 0.003 MMSTB/D, a maximum value of 0.924 MMSTB/D, and a median value of 0.063 MMSTB/D. The total world model has a median value of 0.084 MMSTB/D, which means that 50% of the total world models deviate by less than 84 MSTB/D from the perfect fit. The remaining 50% of the world models have a maximum deviation of 924 MSTB/D from the perfect match, which is less than 1.3% of the total world production. The results of the validation analysis of all investigated countries are presented in Table 2. A thorough inspection of the results of each country reveals that the forecasting models are in very good agreement with the actual production.
4. Discussions and Conclusions

Forecasting is not accomplished by consulting a crystal ball or a mystic of some sort, but by appraising the past, inspecting present conditions, and projecting these into the future based on the best available information. It is well-known that the ultimate oil recovery of any field in the world is only determined when the production management decides to abandon the field for good. This does not occur until the projected oil revenues fall below expected costs and human ingenuity is unable to reverse this relationship. Therefore, it is not a sin to acknowledge that the economic production life of any field is almost impossible to predict efficiently. Hence, forecasts should be flexible to adjustment whenever additional information becomes available or as conditions change. Even though it is inevitable to preclude the possibility of minor inaccurate forecasting results, still it is of paramount importance that the forecast be conducted. Without the forecast, a valid decision or public policy debate on a national or global scale cannot be made.

Several pertinent conclusions can be drawn based on the findings of this investigation:

(1) The multicyclic Hubbert model, which is a modified form of the conventional Hubbert model, has been presented and applied successfully to the prediction of world crude oil production. The multicyclic model has fewer parameters than the original Hubbert model making production data easier and more accurate to fit. Our investigation demonstrated that out of the 47 evaluated countries, 17% have excellent models, 70% have very good models, and 13% have good models.

(2) The OPEC model was developed by combining the individual models of the organization 12 countries. The results of this study estimated OPEC ultimate reserves by 1321 BSTB and the remaining oil by 909 BSTB. OPEC holds about 78% of the world crude oil.

(3) Our study also indicates that OPEC crude oil production will peak in 2026 at a production rate of 53 MMSTB/D. On the basis of 2005 crude oil production rate and recovery techniques, OPEC crude oil reserve is being depleted at an annual rate of 1.25%.

(4) Despite the current world economical crisis, we speculate that OPEC will remain the main world supplier of crude oil up to the end of this century.

(5) Non-OPEC countries have already reached their peak production of 39.6 MMSTB/D in 2006. According to our analysis, the ultimate reserve of these countries is 819 BSTB and their future recoverable oil is 252 BSTB. Non-OPEC countries hold 22% of the world crude oil reserves, which are being depleted at an annual rate of 5.6%.

(6) On the basis of the analysis of all 47 investigated oil producing countries, the results of our study estimated that the world ultimate reserve of crude oil is around 2140 BSTB and that 1161 BSTB are remaining to be produced as of 2005 year end.
(7) The world conventional crude oil production will peak in 2014 at a rate of 79 MMSTB/D. The world crude oil reserves are being depleted at an annual rate of 2.1%.

Appendix

Predicting the Peak Production Using the Method of Inflection Points. Hubbert expressed the production rate \( q(t) \) or \( dQ/dt \) by a parabolic equation in the cumulative production, \( Q \), domain as follows:

\[
q(t) = \frac{dQ}{dt} = aQ + bQ^2
\]  

(A-1)

Since the production rate will be zero when the cumulative production is equal to the ultimate recovery, \( Q_m \), then eq A-1 can be written as follows:

\[
aQ_m + bQ_m^2 = 0
\]  

(A-2)

Solving eq A-2 for the constant \( b \) yields

\[
b = -\frac{a}{Q_m}
\]  

(A-3)

Substituting eq A-3 into eq A-1 and rearranging terms yields

\[
adt = \frac{dQ}{Q - Q_m^2}
\]  

(A-4)

To express the cumulative production as a function of time, the left-hand-side of eq A-4 is integrated from \( t_o \) to \( t \rightarrow t_m \) and the right-hand-side is integrated from \( Q_o \) to \( Q \rightarrow Q_m \). This yields

\[
Q = \frac{Q_m}{1 + D_o e^{-a(t-t_o)}}
\]  

(A-5)

where \( D_o \) is defined as

\[
D_o = \frac{Q_m}{Q_o} - 1
\]  

(A-6)

Equation A-5 is called the logistic equation.

Differentiating eq A-5 with respect to time yields the production rate equation as follows:

\[
q(t) = \frac{Q_o aD_o e^{-a(t-t_o)}}{[1 + D_o e^{-a(t-t_o)}]^2}
\]  

(A-7)

The maximum cumulative production time is obtained by differentiating eq A-1 and setting it to zero

\[
Q_{max} = \frac{Q_o}{2}
\]  

(A-8)

This corresponds to the inflection point on the cumulative production curve. The peak production rate is then calculated by substituting eq A-8 into eq A-1

\[
q_{max} = \frac{a}{4} Q_{max}
\]  

(A-9)

The peak production time \( t_{max} \) can be calculated by transforming the logistic equation, eq A-5, into a linear form and substituting for \( Q_{max} \) at \( t_{max} \) from eq A-8

\[
t_{max} = t_o + \left( \frac{1}{a} \right) \ln(D_o)
\]  

(A-10)

The two inflection points on the bell-shape curve, Hubbert rate/time curve, are determined by taking the second derivative of eq A-7 as follows:  

\[
q''(t) = \frac{Q_o a^2 D_o e^{a(t-t_o)\frac{2}{2}} - 4D_o e^{a(t-t_o)\frac{2}{2}} + e^{2a(t-t_o)}}{[D_o + e^{a(t-t_o)}]^4}
\]  

(A-11)

Setting eq A-11 to zero and solving for time, \( t \), yields the time of inflection points where changes of sign occur on the curve with one complete cycle. The solution gives two inflection points \( t_{inf1} \) and \( t_{inf2} \) defined, respectively, as follows

\[
t_{inf1} = t_o + \left( \frac{1}{a} \right) \ln \left( 2 - \sqrt{3} D_o \right)
\]  

(A-12)

and

\[
t_{inf2} = t_o + \left( \frac{1}{a} \right) \ln \left( 2 + \sqrt{3} D_o \right)
\]  

(A-13)

Substituting either \( t_{inf1} \) or \( t_{inf2} \) into eq A-7 yields the production rate at the inflection points, \( q_{inf} \) as follows:

\[
q_{inf} = \frac{a}{6} Q_{max}
\]  

(A-14)

Using eqs A-9 and A-14, we obtain a relationship between the maximum production rate, \( q_{max} \), and the production rate

<table>
<thead>
<tr>
<th>country</th>
<th>RMSE (MMSTB/D)</th>
<th>country</th>
<th>RMSE (MMSTB/D)</th>
<th>country</th>
<th>RMSE (MMSTB/D)</th>
<th>country</th>
<th>RMSE (MMSTB/D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algeria</td>
<td>0.2967</td>
<td>Argentina</td>
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<td>Equatorial</td>
<td>0.0569</td>
<td>Sudan</td>
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<td>Gabon</td>
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<td>Azerbaijan</td>
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<td>Italy</td>
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<td>Kazakhstan</td>
<td>0.1878</td>
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<td>Canada</td>
<td>0.1266</td>
<td>Malaysia</td>
<td>0.1533</td>
<td>Turkmenistan</td>
<td>0.0784</td>
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<tr>
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<td>China</td>
<td>0.0231</td>
<td>Mexico</td>
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<td>United Kingdom</td>
<td>0.1718</td>
</tr>
<tr>
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<td>Columbia</td>
<td>0.1963</td>
<td>Norway</td>
<td>0.2886</td>
<td>United States</td>
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<td>Oman</td>
<td>0.0554</td>
<td>Uzbekistan</td>
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<td>Denmark</td>
<td>0.0626</td>
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<td>0.0026</td>
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<tr>
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<td>Egypt</td>
<td>0.0216</td>
<td>Russia</td>
<td>0.9237</td>
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<td></td>
</tr>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>minimum</th>
<th>maximum</th>
<th>median</th>
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</thead>
<tbody>
<tr>
<td>OPEC</td>
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<td>0.8211</td>
<td>0.2555</td>
</tr>
<tr>
<td>non-OPEC</td>
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<td>0.9237</td>
<td>0.0626</td>
</tr>
<tr>
<td>total world</td>
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<td>0.9237</td>
<td>0.0840</td>
</tr>
</tbody>
</table>
at the inflection point, \( q_{\text{inf}} \), as follows:

\[
q_{\text{max}} = \frac{3}{2} q_{\text{inf}}
\]  

(A-15)

Nomenclature

\( \alpha \) = constant, eq 1, \( 1/t \), \( 1/\text{year} \)

\( BSTB \) = billion stock tank barrels

\( CV \) = coefficient of variation, eq 7, dimensionless

\( D_o \) = dimensionless cumulative factor

\( F(CV) \) = cumulative probability function, eq 8

\( k \) = number of production cycles, eq 1

\( MSTB/D \) = thousand stock tank barrels per day

\( MMSTB/D \) = million stock tank barrels per day

\( n \) = number of observations, eq 6, or countries, eq 8

\( N_{\text{pa}} \) = total ultimate oil recovery, eq 3, MMSTB

\( N_{\text{FR}} \) = future recoverable oil, eq 5, MMSTB

\( q(t) \) = production rate, MMSTB/D

\( q_{\text{act}} \) = actual production rate, MMSTB/D

\( q_{\text{cal}} \) = calculated production rate, MMSTB/D

\( q_{\text{inf}} \) = oil production rate at the inflection point, MMSTB/D

\( q_{\text{max}} \) = peak or maximum oil production rate of each cycle, MMSTB/D

\( Q_{\infty} \) = ultimate oil recovery, eq 2, MMSTB

\( r \) = country’s rank in the goodness of fit of the production trend, eq 8

\( \text{RMSE} \) = root mean squares of the errors, eq 6, MMSTB/D

\( t \) = time, calendar year

\( t_{\text{inf}} \) = inflection point time, calendar year

\( t_{\text{max}} \) = peak production time, calendar year

\( t_0 \) = arbitrary time, calendar year

\( TSTB \) = trillion stock tank barrels

\( \sigma \) = standard deviation of the dimensionless root mean squares of the errors

\( \mu \) = mean of the dimensionless root mean squares of the errors